B.K. BIRLA CENTRE FOR EDUCATION

SARALA BIRLA GROUP OF SCHOOLS
A CBSE DAY-CUM-BOYS' RESIDENTIAL SCHOOL

POST MID TERM (2025-26)
MATHEMATICS-MARKING KEY

Class: XI — **Time: 1hrs**
Date: 07-01-26 — **Max Marks: 25**
Admission no: — **Roll no:**

General Instructions:
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 6 MCQs carrying 1 mark each
3. Section B has 2 questions carrying 02 marks each.
4. Section C has 2 questions carrying 03 marks each.
5. Section D has 1 question carrying 05 marks each.
6. Section E has 1 case-based integrated units of assessment (04 marks each) with sub-parts.
7. All Questions are compulsory.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

1. **Q1.** Find the value of: 1M

$$\lim_{y \to 2} \frac{y^2 - 4}{y - 2}$$

a) 2
b) 4
c) 1
d) 0
Answer: (b) 4
Solution:

$$\frac{y^2 - 4}{y - 2} = \frac{(y-2)(y+2)}{y - 2} = y + 2 \Rightarrow 2 + 2 = 4$$

2. **Q2.** Find the value of: 1M

$$\lim_{y \to \infty} \frac{2}{y}$$

a) 0
b) 1
c) 2
d) Infinity
Answer: (a) 0
Solution:
As $y \to \infty, \frac{2}{y} \to 0$.

3. Find the value of: 1M

$$\lim_{x \to 4} \frac{x^2 - 2x - 8}{x - 4}$$

a) 0
b) 2
c) 8
d) 6
Answer: (d) 6
Solution:

$$x^2 - 2x - 8 = (x-4)(x+2) \Rightarrow \frac{(x-4)(x+2)}{x-4} = x + 2 \Rightarrow 4 + 2 = 6$$

4. Find the value of: 1M

$$\lim_{x \to 3} \frac{x^2 - 9}{x - 3}$$

a) 0
b) 3
c) Infinity
d) 6
Answer: (d) 6
Solution:

$$x^2 - 9 = (x-3)(x+3) \Rightarrow \frac{(x-3)(x+3)}{x-3} = x + 3 \Rightarrow 3 + 3 = 6$$

5. Find the value of: 1M

$$\lim_{x \to \infty} \frac{x^2 - 9}{x^2 - 3x + 2}$$

a) 1
b) 2
c) 0
d) Limit does not exist
Answer: (a) 1
Solution:
For large x, divide numerator and denominator by x^2:

$$\frac{x^2 - 9}{x^2 - 3x + 2} = \frac{1 - \frac{9}{x^2}}{1 - \frac{3}{x} + \frac{2}{x^2}} \Rightarrow \frac{1 - 0}{1 - 0 + 0} = 1$$

6. **Which of the following limits does NOT yield 1?** 1M
a) $\lim_{x \to 0} 1$
b) $\lim_{x \to \infty} (x^{-2} + x^{-1} + 1)$
c) $\lim_{x \to \infty} \frac{1}{e^x} + 1$
d) $\lim_{x \to \infty} \frac{x^3 + x^2 + 32x + 1}{x^2 - 3x + 2}$

Answer: ✓ Option (d)

Solution:
- **Option (a):**

$$\lim_{x \to 0} 1 = 1$$

- **Option (b):**

$$\lim_{x\to\infty}(x^{-2}+x^{-1}+1)=0+0+1=1$$

- **Option (c):**

$$\lim_{x\to\infty}\frac{1}{e^x}+1=0+1=1$$

- **Option (d):**

Highest power in numerator = x^3
Highest power in denominator = x^2
As $x\to\infty$:

SECTION B

7. **Find the limits:** 2M
(i) $\lim_{x\to 1}(x^3-x^2+1)$
(ii) $\lim_{x\to 3}[x(x+1)]$

Solution:

(i) $\boldsymbol{\lim_{x\to 1}(x^3-x^2+1)}$
Since the function is a polynomial, substitute $x=1$:

$$=1^3-1^2+1=1-1+1=1$$

✓ **Answer: 1** 1M

(ii) $\boldsymbol{\lim_{x\to 3}[x(x+1)]}$
Again, substitute $x=3$:

$$=3(3+1)=3\times 4=12$$

✓ **Answer: 12** 1M

8. Find the derivative of the function $f(x)=2x^2+3x-5$at $x=-1$. Also prove that $f'(0)+3f'(-1)=0$. 2M

Solution:
Given:

$$f(x)=2x^2+3x-5$$

Differentiate:

$$f'(x)=\frac{d}{dx}(2x^2)+\frac{d}{dx}(3x)-\frac{d}{dx}(5)=4x+3$$

Derivative at $\boldsymbol{x=-1}$:

$$f'(-1)=4(-1)+3=-4+3=-1$$

1M

✓ **Answer:** $f'(-1)=-1$

Verification: $\boldsymbol{f'(0)+3f'(-1)=0}$

$$f'(0)=4(0)+3=3$$
$$f'(0)+3f'(-1)=3+3(-1)=3-3=0$$

✓ **Verified:** $0=0$

1M

SECTION C

9. **Find the derivative of $\cos x$ from first principle.** 3M

Solution:
Using first principle,

$$\frac{d}{dx}(\cos x) = \lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

Apply trigonometric identity:

$$\cos(x+h) = \cos x\cos h - \sin x\sin h$$

So,

$$\frac{d}{dx}(\cos x) = \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$$

Factor out $\cos x$:

$$= \lim_{h\to 0}[\cos x\frac{\cos h - 1}{h} - \sin x\frac{\sin h}{h}]$$

2M

Use limits:

$$\lim_{h\to 0}\frac{\sin h}{h} = 1, \lim_{h\to 0}\frac{\cos h - 1}{h} = 0$$

Therefore,

$$= \cos x(0) - \sin x(1)$$
$$= -\sin x$$

Answer: $\boxed{-\sin x}$ 1M

10. **If the function $f(x)$ satisfies $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$, evaluate $\lim_{x\to 1} f(x)$.** 3M

Solution:
Given:

$$\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$$

As $x \to 1$, the denominator:

$$x^2 - 1 = (x-1)(x+1) \to 0$$

2M

For the limit to exist and not be infinite, the numerator must also approach 0:

$$f(x) - 2 \to 0 \Rightarrow f(x) \to 2$$

Therefore:

$$\lim_{x\to 1} f(x) = 2$$

1M

Answer: $\boxed{2}$

SECTION D

11. **Evaluate** 5M

(a) Let $a_1, a_2, \dots, a_n$ be real numbers and define

$$f(x) = (x-a_1)(x-a_2)\cdots(x-a_n).$$

Evaluate

$$\lim_{x\to a_k} f(x) \text{for some } a_k \in \{a_1, a_2, \dots, a_n\}.$$

(b) The function $f(x)$is defined as

$$f(x) = \begin{cases} mx + n, & 0 \leq x \leq 1 \\ nx^3 + m, & x > 1 \end{cases}$$

For what integer values of $m$and $n$does the limit
$\lim_{x \to 1} f(x)$exist?

Answer Key / Marking Scheme

(a) Solution (2 marks):
At $x = a_k$, only the factor $(x - a_k)$becomes zero, while all other factors remain finite.
So,

$$\lim_{x \to a_k} f(x) = 0$$

2M

✓ **Final Answer:** $\boxed{0}$

(b) Solution (3 marks):
For limit to exist at $x = 1$, left-hand and right-hand limits must be equal.
Left-limit:

$$f(1^-) = m(1) + n = m + n$$

Right-limit:

$$f(1^+) = n(1)^3 + m = n + m$$

Since both expressions are equal automatically:

$$m + n = m + n$$

This is always true. 3M

✓ So, the limit exists **for all integer values of** m**and** n.

➡ **Final Answer: The limit exists for all integers** m**and** n**.**

SECTION E

12. A toy manufacturing company programs a cutting machine that works using mathematical functions. 4M
The first cutting program uses the function:

$$P(x) = \frac{x^{15} - 1}{x^{10} - 1}$$

To calibrate the machine, the output must be known as $x$approaches 1.
Answer the following:
(a) State the algebraic identity used to factor the expressions $x^{15} - 1$and $x^{10} - 1$.
(b) Evaluate: $\lim_{x \to 1} P(x)$.
(c) Later, the company updates the program to use another expression:

$$Q(x) = 1 + x + x^2 + \cdots + x^{10}$$

Find: $\lim_{x \to 1} Q(x)$.

Solution (Marking Scheme)

(a) Algebraic Pattern (1 mark)

$$x^n - 1 = (x - 1)(1 + x + x^2 + \cdots + x^{n-1})$$

✓ **Pattern used: Difference of powers identity.** 1M

(b) Evaluate $\lim_{x \to 1} P(x)$**(2 marks)**

$$P(x) = \frac{x^{15} - 1}{x^{10} - 1} = \frac{(x-1)(1 + x + x^2 + \cdots + x^{14})}{(x-1)(1 + x + x^2 + \cdots + x^9)}$$

Cancel $(x - 1)$:

$$P(x) = \frac{1 + x + x^2 + \cdots + x^{14}}{1 + x + x^2 + \cdots + x^9}$$

Now apply $x \to 1$:

- Numerator has **15 terms**, each becomes 1 → total = **15** 2M
- Denominator has **10 terms**, each becomes 1 → total = **10**

$$\lim_{x \to 1} P(x) = \frac{15}{10} = \frac{3}{2}$$

✓ **Answer:** $\boxed{\frac{3}{2}}$

(c) Evaluate $\lim_{x\to 1} Q(x)$(2 marks)

$$Q(x) = 1 + x + x^2 + \cdots + x^{10}$$

As $x \to 1$, every term becomes 1:

$$Q(1) = 11$$

1M

✓ **Answer:** $\boxed{11}$

*******************BEST OF LUCK******************